國立臺灣大學100學年度轉學生招生考試試題

題號： 18
科目：微積分(A)

1. (20%) Let $f(x) = \frac{\sqrt{x}}{x^2+1}$, $x \geq 0$ and $F'(x) = f(x)$. Prove that $|F(x^2) - F(y^2)| \leq |x-y|$ for all $x, y \geq 0$.

2. (20%) Let $S_n = \sum_{k=1}^{k=n} \frac{\sqrt{k}}{n^{\frac{3}{2}}}$. Find $\lim_{n\to\infty} S_n$.

3. (20%) Let $S_m = \sum_{n=1}^{n=m} (-1)^{n+1} a_n$, $m = 1, 2, \cdots$. Suppose i) $a_n \geq 0$ for all n ii) $a_n \geq a_{n+1}$ for all n iii) $\lim_{n\to\infty} a_n = 0$. Prove that a) $S_{2m+2} \geq S_{2m}$ for all m b) $S_{2m} \leq a_1$ for all m c) $\lim_{m\to\infty} S_m$ exists.

4. (20%) Let Ω be the entire xy plane. Evaluate the double integral

$$\iint_{\Omega} e^{-(x^2+2xy+5y^2)} dxdy.$$

5. (20%) Evaluate the following line integral traced in the counterclockwise direction

$$\int_C \left(\frac{-y}{x^2+y^2}\right)dx + \left(\frac{x}{x^2+y^2}\right)dy$$

where C is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

試題隨卷繳回